1.1 Identify Points, Lines, and Planes

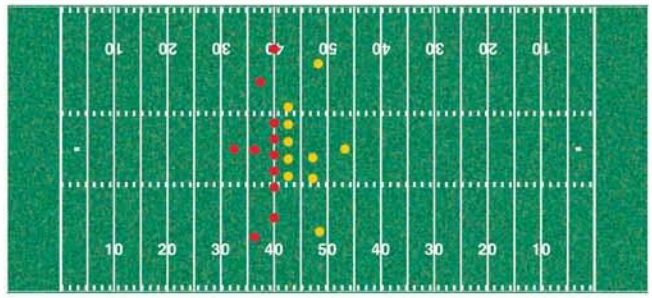
Before You studied basic concepts of geometry.

Now You will name and sketch geometric figures.

Why So you can use geometry terms in the real world, as in Ex. 13.

Key Vocabulary
- **undefined terms**
  point, line, plane
- **collinear points**
- **coplanar points**
- **defined terms**
- **line segment**
- **endpoints**
- **ray**
- **opposite rays**
- **intersection**

In the diagram of a football field, the positions of players are represented by *points*. The yard lines suggest *lines*, and the flat surface of the playing field can be thought of as a *plane*.



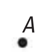
In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

TAKE NOTES
When you write new concepts and yellow-highlighted vocabulary in your notebook, be sure to copy all associated diagrams.

KEY CONCEPT — *For Your Notebook*

Undefined Terms

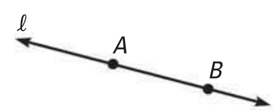
Point A **point** has no dimension. It is represented by a dot.



point *A*

Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

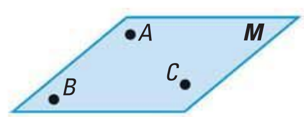
Through any two points, there is exactly one line. You can use any two points on a line to name it.



line ℓ, line *AB* ($\overleftrightarrow{AB}$), or line *BA* ($\overleftrightarrow{BA}$)

Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



plane *M* or plane *ABC*

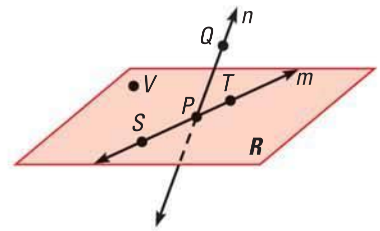
Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

EXAMPLE 1 Name points, lines, and planes

VISUAL REASONING
There is a line through points *S* and *Q* that is not shown in the diagram. Try to imagine what plane *SPQ* would look like if it were shown.

a. Give two other names for $\overleftrightarrow{PQ}$ and for plane *R*.

b. Name three points that are collinear.
Name four points that are coplanar.



Solution

a. Other names for $\overleftrightarrow{PQ}$ are $\overleftrightarrow{QP}$ and line *n*. Other names for plane *R* are plane *SVT* and plane *PTV*.

b. Points *S*, *P*, and *T* lie on the same line, so they are collinear. Points *S*, *P*, *T*, and *V* lie in the same plane, so they are coplanar.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Example 1

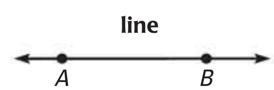
1. Use the diagram in Example 1. Give two other names for $\overleftrightarrow{ST}$. Name a point that is *not* coplanar with points *Q*, *S*, and *T*.

DEFINED TERMS In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

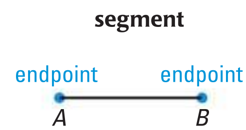
KEY CONCEPT — *For Your Notebook*

Defined Terms: Segments and Rays

Line *AB* (written as $\overleftrightarrow{AB}$) and points *A* and *B* are used here to define the terms below.

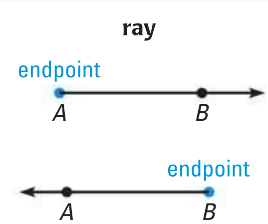


Segment The **line segment** *AB*, or **segment** *AB*, (written as $\overline{AB}$) consists of the **endpoints** *A* and *B* and all points on $\overleftrightarrow{AB}$ that are between *A* and *B*. Note that $\overline{AB}$ can also be named $\overline{BA}$.



Ray The **ray** *AB* (written as $\overrightarrow{AB}$) consists of the endpoint *A* and all points on $\overleftrightarrow{AB}$ that lie on the same side of *A* as *B*.

Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.



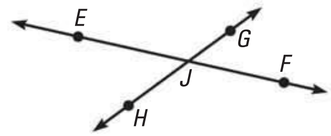
If point *C* lies on $\overleftrightarrow{AB}$ between *A* and *B*, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are **opposite rays**.

Segments and rays are collinear if they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar if they lie in the same plane.

EXAMPLE 2 Name segments, rays, and opposite rays

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint J. Which of these rays are opposite rays?



AVOID ERRORS

In Example 2, $\overrightarrow{JG}$ and $\overrightarrow{JF}$ have a common endpoint, but are not collinear. So they are *not* opposite rays.

Solution

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint J are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$, and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint J are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, and $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

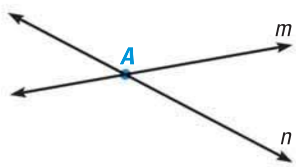
✓ GUIDED PRACTICE for Example 2

Use the diagram in Example 2.

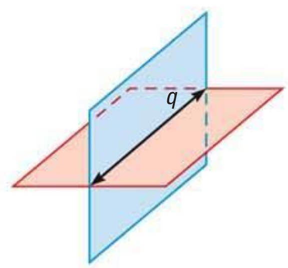
2. Give another name for $\overline{EF}$.

3. Are $\overrightarrow{HJ}$ and $\overrightarrow{JH}$ the same ray? Are $\overrightarrow{HJ}$ and $\overrightarrow{HG}$ the same ray? *Explain*.

INTERSECTIONS Two or more geometric figures *intersect* if they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of two different lines is a point.



The intersection of two different planes is a line.

EXAMPLE 3 Sketch intersections of lines and planes

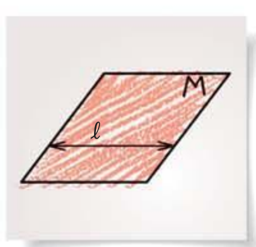
a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

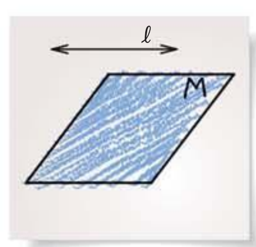
c. Sketch a plane and a line that intersects the plane at a point.

Solution

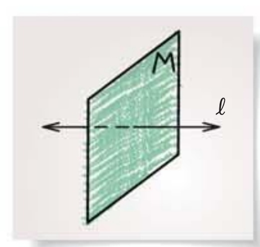
a.



b.



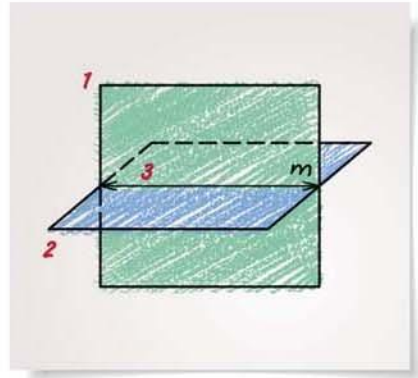
c.



EXAMPLE 4 Sketch intersections of planes

Sketch two planes that intersect in a line.

Solution



STEP 1 **Draw** a vertical plane. Shade the plane.

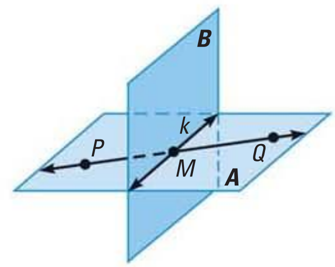
STEP 2 **Draw** a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

STEP 3 **Draw** the line of intersection.

✓ GUIDED PRACTICE for Examples 3 and 4

4. Sketch two different lines that intersect a plane at the same point.

Use the diagram at the right.



5. Name the intersection of $\overleftrightarrow{PQ}$ and line k.
6. Name the intersection of plane A and plane B.
7. Name the intersection of line k and plane A.

1.1 EXERCISES

HOMEWORK KEY

○ **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 19, and 43

★ **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 13, 16, and 43

SKILL PRACTICE

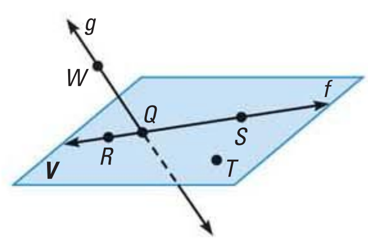
1. **VOCABULARY** Write in words what each of the following symbols means.

   a. Q b. $\overline{MN}$ c. $\overrightarrow{ST}$ d. $\overleftrightarrow{FG}$

2. ★ **WRITING** *Compare* collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? *Explain.*

EXAMPLE 1 on p. 3 for Exs. 3–7

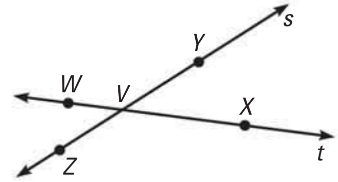
NAMING POINTS, LINES, AND PLANES **In Exercises 3–7, use the diagram.**



3. Give two other names for $\overleftrightarrow{WQ}$.
4. Give another name for plane V.
5. Name three points that are collinear. Then name a fourth point that is *not* collinear with these three points.
6. Name a point that is *not* coplanar with R, S, and T.
7. ★ **WRITING** Is point W coplanar with points Q and R? *Explain.*

EXAMPLE 2
on p. 4
for Exs. 8–13

NAMING SEGMENTS AND RAYS **In Exercises 8–12, use the diagram.**



8. What is another name for $\overline{ZY}$?

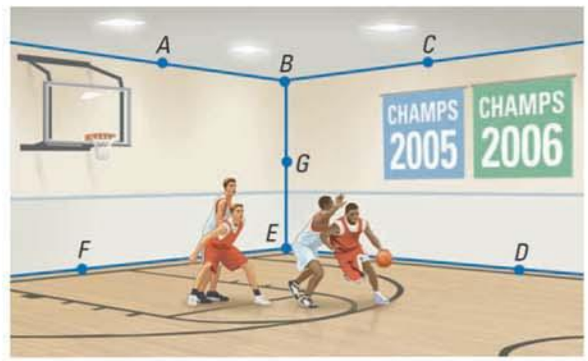
9. Name all rays with endpoint *V*.

10. Name two pairs of opposite rays.

11. Give another name for $\overrightarrow{WV}$.

12. **ERROR ANALYSIS** A student says that $\overrightarrow{VW}$ and $\overrightarrow{VZ}$ are opposite rays because they have the same endpoint. *Describe* the error.

13. ★ **MULTIPLE CHOICE** Which statement about the diagram at the right is true?



(A) *A*, *B*, and *C* are collinear.

(B) *C*, *D*, *E*, and *G* are coplanar.

(C) *B* lies on $\overrightarrow{GE}$.

(D) $\overrightarrow{EF}$ and $\overrightarrow{ED}$ are opposite rays.

EXAMPLES 3 and 4
on pp. 4–5
for Exs. 14–23

SKETCHING INTERSECTIONS **Sketch the figure described.**

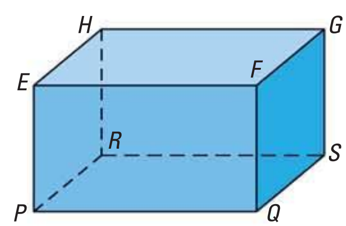
14. Three lines that lie in a plane and intersect at one point

15. One line that lies in a plane, and one line that does not lie in the plane

16. ★ **MULTIPLE CHOICE** Line *AB* and line *CD* intersect at point *E*. Which of the following are opposite rays?

(A) $\overrightarrow{EC}$ and $\overrightarrow{ED}$ (B) $\overrightarrow{CE}$ and $\overrightarrow{DE}$ (C) $\overrightarrow{AB}$ and $\overrightarrow{BA}$ (D) $\overrightarrow{AE}$ and $\overrightarrow{BE}$

READING DIAGRAMS **In Exercises 17–22, use the diagram at the right.**



17. Name the intersection of $\overleftrightarrow{PR}$ and $\overleftrightarrow{HR}$.

18. Name the intersection of plane *EFG* and plane *FGS*.

19. Name the intersection of plane *PQS* and plane *HGS*.

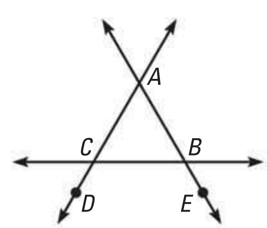
20. Are points *P*, *Q*, and *F* collinear? Are they coplanar?

21. Are points *P* and *G* collinear? Are they coplanar?

22. Name three planes that intersect at point *E*.

23. **SKETCHING PLANES** Sketch plane *J* intersecting plane *K*. Then draw a line ℓ on plane *J* that intersects plane *K* at a single point.

24. **NAMING RAYS** Name 10 different rays in the diagram at the right. Then name 2 pairs of opposite rays.



25. **SKETCHING** Draw three noncollinear points *J*, *K*, and *L*. Sketch $\overline{JK}$ and add a point *M* on $\overline{JK}$. Then sketch $\overrightarrow{ML}$.

26. **SKETCHING** Draw two points *P* and *Q*. Then sketch $\overrightarrow{PQ}$. Add a point *R* on the ray so that *Q* is between *P* and *R*.

○ = WORKED-OUT SOLUTIONS ★ = STANDARDIZED TEST PRACTICE

REVIEW ALGEBRA
For help with equations of lines, see p. 878.

ALGEBRA In Exercises 27–32, you are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

27. $y = x - 4$; $A(5, 1)$
28. $y = x + 1$; $A(1, 0)$
29. $y = 3x + 4$; $A(7, 1)$
30. $y = 4x + 2$; $A(1, 6)$
31. $y = 3x - 2$; $A(-1, -5)$
32. $y = -2x + 8$; $A(-4, 0)$

GRAPHING Graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*.

33. $x \le 3$
34. $x \ge -4$
35. $-7 \le x \le 4$
36. $x \ge 5$ or $x \le -2$
37. $x \ge -1$ or $x \le 5$
38. $|x| \le 0$

39. CHALLENGE Tell whether each of the following situations involving three planes is possible. If a situation is possible, make a sketch.

a. None of the three planes intersect.

b. The three planes intersect in one line.

c. The three planes intersect in one point.

d. Two planes do not intersect. The third plane intersects the other two.

e. Exactly two planes intersect. The third plane does not intersect the other two.

PROBLEM SOLVING

EXAMPLE 3
on p. 4 for Exs. 40–42

EVERYDAY INTERSECTIONS What kind of geometric intersection does the photograph suggest?

40.

41.

42.

43. ★ **SHORT RESPONSE** *Explain* why a four-legged table may rock from side to side even if the floor is level. Would a three-legged table on the same level floor rock from side to side? Why or why not?

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44. SURVEYING A surveying instrument is placed on a tripod. The tripod has three legs whose lengths can be adjusted.

a. When the tripod is sitting on a level surface, are the tips of the legs coplanar?

b. Suppose the tripod is used on a sloping surface. The length of each leg is adjusted so that the base of the surveying instrument is level with the horizon. Are the tips of the legs coplanar? *Explain*.

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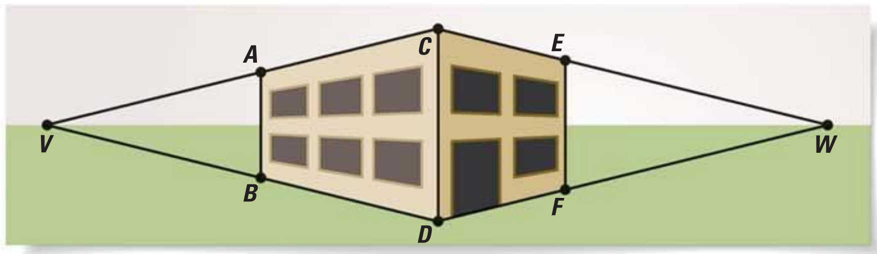
45. MULTI-STEP PROBLEM In a *perspective drawing*, lines that do not intersect in real life are represented by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*. The diagram shows a drawing of a house with two vanishing points.



a. Trace the black line segments in the drawing. Using lightly dashed lines, join points A and B to the vanishing point W. Join points E and F to the vanishing point V.

b. Label the intersection of $\overleftrightarrow{EV}$ and $\overleftrightarrow{AW}$ as G. Label the intersection of $\overleftrightarrow{FV}$ and $\overleftrightarrow{BW}$ as H.

c. Using heavy dashed lines, draw the hidden edges of the house: $\overline{AG}$, $\overline{EG}$, $\overline{BH}$, $\overline{FH}$, and $\overline{GH}$.

46. CHALLENGE Each street in a particular town intersects every existing street exactly one time. Only two streets pass through each intersection.

2 streets | 3 streets | 4 streets

a. A traffic light is needed at each intersection. How many traffic lights are needed if there are 5 streets in the town? 6 streets?

b. *Describe* a pattern you can use to find the number of additional traffic lights that are needed each time a street is added to the town.

MIXED REVIEW

Find the difference. *(p. 869)*

47. $-15 - 9$ **48.** $6 - 10$ **49.** $-25 - (-12)$

50. $13 - 20$ **51.** $16 - (-4)$ **52.** $-5 - 15$

PREVIEW
Prepare for Lesson 1.2 in Exs. 53–58.

Evaluate the expression. *(p. 870)*

53. $5 \cdot |-2 + 1|$ **54.** $|-8 + 7| - 6$ **55.** $-7 \cdot |8 - 10|$

Plot the point in a coordinate plane. *(p. 878)*

56. $A(2, 4)$ **57.** $B(-3, 6)$ **58.** $E(6, 7.5)$